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Equation of state of the trans-Planckian dark energy and the coincidence problem

Mar Bastero-Gil*
Centre for Theoretical Physics, University of Sussex, Falmer, Brighton BN1 9QJ, United Kingdom

Laura Mersini†
Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy

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Observational evidence suggests that our Universe is presently dominated by a dark energy component and is undergoing accelerated expansion. We recently introduced a model, motivated by string theory for short-distance physics, for explaining dark energy without appealing to any fine tuning. The idea of trans-Planckian dark energy (TDE) was based on the freeze-out mechanism of the ultralow frequency modes, $\omega(k)$, of very short distances, by the expansion of the background universe, $\omega(k) \leqslant H$. In this paper we address the issue of the stress-energy tensor for nonlinear short-distance physics and explain the need to modify Einstein equations in this regime. From the modified Einstein equations we then derive the equation of state for the TDE model, which has the distinctive feature of being continually time dependent. The explanation of the coincidence puzzle relies entirely on the intrinsic time evolution of the TDE equation of state.

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I. INTRODUCTION

Cosmological observations of large scale structure, supernova type 1a age of the Universe, and cosmic microwave background (CMB) data strongly indicate that the universe is dominated by a dark energy component with negative pressure [1]. In addition to the difficulty of coming up with a natural explanation for the smallness of the observed dark energy, an equal challenge is the "cosmic coincidence" problem.

Recently we proposed a model [2] for explaining the observed dark energy without appealing to fine-tuning or anthropic arguments. This model is based on the nonlinear behavior of trans-Planckian metric perturbation modes which was motivated by closed string theory [3,4] and quantum gravity [5]. The trans-Planckian dark energy (TDE) model was based on the freeze-out mechanism of the short-distance modes with ultralow energy by the expansion of the background universe, H, and it naturally explained the smallness of the observed dark energy.

In this paper we study the stress-energy tensor of the TDE model in order to calculate the equation of state for these short-distance stringy modes. As we will show, the frozen tail modes start having a negative pressure of the same order as their positive energy density soon after the matter domination era. Thus it is only at low redshifts that they become important for driving the universe into an accelerated expansion and dominate the Hubble expansion rate H. A distinctive feature of the TDE model is that its equation of state, w_H, is always strongly time dependent at any epoch in the evolution of the universe (e.g., $w_H = -1/3$ during the radiation dominated era but it becomes $w_H = -1/2$ at matter domination). It becomes increasingly negative at later times until it approaches the limiting value $w_H = -1$, after the matter domination time, t_{eq}.

The calculation of the components of the stress-energy tensor, $T_{\mu\nu}$, namely, the pressure and energy density, is given in Sec. II. Due to the nonlinearity of short-distance physics, the Bianchi identity is generically violated for all these models. Therefore one needs to modify the Einstein equations, ($T_{\mu\nu}$), such that the modified ones satisfy the Bianchi identity. From physical considerations, the need for modifying Einstein equations in the nonlinear regime of short-distance physics is to be expected, due to nonequilibrum dynamics of the short-distance modes. In practical terms this is not easy to carry out in an unambiguous way, for a simple reason: we do not have a unique effective theory valid at trans-Planckian energies or a Lagrangian description of the theory in this regime [6]. The only information available to most trans-Planckian models [7,2,8–12] is the field equation of motion (with a few exceptions, see Ref. [13]). Nevertheless all these models do violate the Bianchi identity and the energy conservation law, if $T_{\mu\nu}$ is not modified accordingly.

Based on the equation of motion as our sole information for short-distance physics, we therefore use a kinetic theory approach [14] for modifying Einstein equations in the absence of an effective Lagrangian description. The assumption made is that a kinetic theory description of the cosmological fluid is valid even in the trans-Planckian regime. Despite its nonlinear behavior at short distances, this imperfect fluid shares the same symmetries, namely, homogeneity and isotropy, as the background Friedmann-Robertson-Walker (FRW) universe. Then the corrections $\tau_{\mu\nu}$ to the stress-energy tensor $T_{\mu\nu}$ will also be of the diagonal form [15]

$$\tau_{\mu\nu} = (\overline{\epsilon} + \Pi) u_\mu u_\nu + \Pi g_{\mu\nu}. \tag{1}$$

In Sec. III we explore the observational consequences of the model with the puzzle of "cosmic coincidence" in mind. A summary is given in Sec. IV. A discussion of the nonequili-

*Email address: mbg20@pact.cpes.susx.ac.uk
†Email address: l.mersini@sns.it

brum dynamics and distribution function for the trans-Planckian (TD) modes, as well as details of averaging their energy and pressure, are presented in the Appendix.

II. THE EQUATION OF STATE FROM THE MODIFIED EINSTEIN EQUATIONS

A. Analytical expression for $T_{\mu\nu}$

Trans-Planckian models that investigate the sensitivity of the cosmic microwave background (CMB) spectrum or Hawking radiation to short-distance physics, all introduce a nonlinear, time-dependent frequency for the very short wavelength modes [2,7–10]:

$$\omega[p]=f[p]=f[k/a]. \tag{2}$$

The physical momentum p is related to the comoving wave number k by $p=k/a$, with a the scale factor. Most of these models lack a Lagrangian description, and all the information they propose about short-distance physics is contained in the mode equation of motion:[1]

$$[\Box+\omega(k,a)^2]\phi_k=0. \tag{3}$$

The expectation that Einstein equations will not hold unless they are modified in the nonlinear regime of short-distance physics is fully reasonable and it is based on the fact that the Bianchi identity and energy conservation law will be violated due to the nonlinear time dependence of ω. In terms of kinetic theory, the time dependence of the group velocity v_g indicates departure from equilibrium [16] (see the Appendix). Here we study the modifications of $T_{\mu\nu}$ for a specific class, the TDE model [2]. Our approach is based on kinetic theory and the pressure modifications are obtained through balance equations.

In the TDE model we are considering, the dispersed frequency for short-distance metric perturbation modes is

$$\omega^2[p]=p^2\mathcal{E}[p/p_c]=p^2\left[\frac{\epsilon_1}{1+u}+\frac{\epsilon_3 u}{(1+u)^2}\right], \tag{4}$$

$$u=\exp[2p/p_c], \tag{5}$$

where p_c is of order of the Planck mass or string scale M, p is the physical momentum, and ϵ_i are arbitrary constants. The maximum of $\omega[p]$ is around $p\approx p_c$. The frequency function behaves as

$$\omega^2[p]\approx p^2[1+O(p^2/M^2)],\quad p\ll M, \tag{6}$$

for the modes in the sub-Planckian regime, and as

$$\omega^2[p]\approx\sqrt{\epsilon_1+\epsilon_3}p^2\exp[-2p/M],p\gg M, \tag{7}$$

for those modes in the TP regime. The nonlinear exact function Eq. (4) for the frequency can be fitted to $\omega[p]^2\sim p^2/(\cosh[p/p_c-1]^2)$.

Lorentz invariance is broken due to the nonlinearity at short distances. Therefore, the *fixed* cutoff scale $p_c=M$, together with all the trans-Planckian modes pick a *preferred frame*, the CMB frame. This frame is freely falling along the comoving geodesics, with respect to the physical FRW Universe.[2] Sometimes we will refer to trans-Planckian modes as the modes inside a small box with fixed Planck size, $l_p=1/M$, in the preferred frame, since their wavelength $\lambda_{TP}<l_p$ is smaller that the "size of the box," $p>M$. In this picture, Lorentz invariance is broken in the small box but restored in the large box with size $L=a/M$, i.e, the Universe. Thus the physical momenta modes for the "small box" bound observers in the preferred frame are the comoving wave number modes for the "outside" observers, in the Lorentz invariant FRW Universe, that "see" the preferred frame in a free fall.

Let us first address the issue of how the energy density components behave with time, prior to the pressure modifications. We will refer to the wave packets of the modes centered around a momentum p_i as particles. Then, their group velocity $v_g=d\omega/dp$ is time dependent through its nonlinear p dependence, and is different from the phase velocity, $v_c=\omega/p$. Therefore, the short-distance modes are out of thermal equilibrium, due to their nonlinear frequency and group velocity $v_g\neq 1$. Meanwhile a thermal state is restored at large scales, $(\lambda\gg l_P)$, where the frequency is nearly linear and thus $v_g\simeq 1$. Thus we need to average the contribution of the short-distance modes to the energy and pressure in the Universe, over many of their wavelengths, in order to obtain an effective large scale thermal state. That is why in obtaining the equation of state $\langle w_i\rangle$ for the trans-Planckian modes, prior to the pressure modifications Π_i, the averaging is done in time scales of cosmological order. Details of averaging are provided in the Appendix. The equation of state $\langle w_i\rangle$ prior to viscous pressure modifications is obtained from the expression $\langle w_i\rangle=\langle\overline{p_i}\rangle/\langle\rho_i\rangle$ with $\langle\overline{p_i}\rangle=-\langle\rho_i\rangle-\langle(a/3)d\rho_i/da\rangle$, with a the scale factor. Based on the behavior of v_g with p, we divide the dispersion function into four regions (see Fig. 1):

Region 0: Linear regime, up to $p\approx p_c$, such that $\omega[p]\simeq p$. These modes behave as radiation, Eq. (6), with the averaged pressure expression $\overline{p_0}=\rho_0/3$.

Region I: Around the maximum of the dispersion function, up to some value $p_B>M$ in physical momentum, where ω can be expanded in a polynomial series

$$\omega[p]\approx p[a_0+a_1(p/M)+a_2(p/M)^2+\cdots], \tag{8}$$

where a_i are constants $(a_2<0)$. Region I is dual to region 0. We use Eq. (8) to estimate the energy density

[1]The ω^2 term collectively denotes the generalized frequency that appears as a mass squared term in the equation. Depending in the particular problem studied it may also include other terms such as, for example, the coupling of the modes to the curvature of the universe, a''/a, if the equation under consideration is that of metric perturbations.

[2]See Ref. [16] for a very nice treatment of issues related to a fixed physical cutoff in a preferred frame.

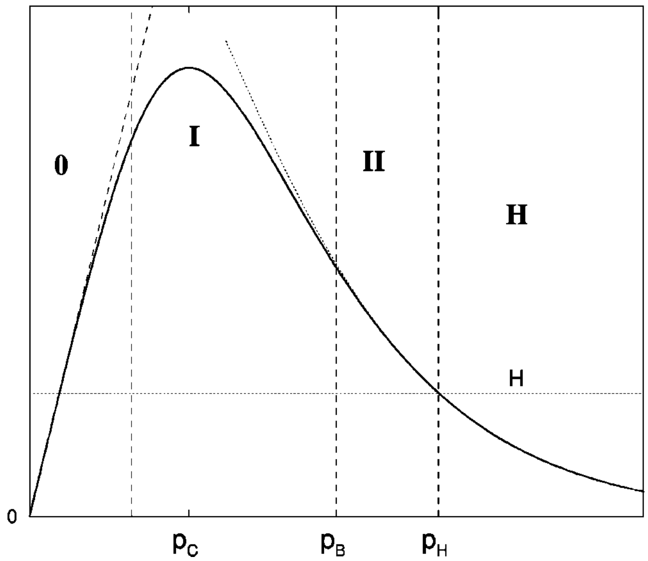

FIG. 1. The dispersion function for the frequency $\omega[p]$ vs p. The separation into four regions is based on the behavior of the group velocity. The "tail" is denoted by region "H."

$$\rho_I \simeq \frac{C}{a^4}\int_M^{k_B} dk k^3\left[a_0 + a_1\frac{k}{aM} + a_2\left(\frac{k}{aM}\right)^2 + \cdots\right]$$
$$\simeq \frac{CM^4}{a^4}\left[\frac{a_0}{4}(x_B^4-1)+\frac{a_1}{5a}(x_B^5-1)+\cdots\right] \propto \frac{M^4}{a^4}, \quad (9)$$

where $x_B = k_B/M > 1$, $\langle x_B\rangle = O(1)$. The constant $C = |\beta_k|^2/(2\pi^2)$ denotes the Bogoliubov coefficient squared, which in our model does not depend on the wave number k [2], with $|\beta_k|^2 \approx \exp(-4\pi\sqrt{\epsilon_1})$. Therefore, ρ_I behaves like radiation plus $O(1/a^2)$ corrections in its averaged equation of state, $\langle \overline{p_I}\rangle = (1/3 - A/a^2 + \cdots)\langle\rho_I\rangle$. Regions I and 0 contribute to the radiation energy component in the Universe.

Region II: From some mode $p_B \gg M$ onwards, defined such that its frequencies can be best fitted to an exponential dependence on p, $\omega[p] \approx p\exp[-p/M]$. The energy density for this region is

$$\rho_{II} \simeq \frac{C}{a^4}\int_{k_B}^{k_H} dk k^3 \exp[-2k/aM] = C\frac{M^4}{a^3}(F[x_B] - F[x_H]), \quad (10)$$

where

$$F[x_i] = \left(\frac{x_i^3}{2} + \frac{3x_i^2}{4} + \frac{3x_i}{4} + \frac{3}{8}\right)\exp[-2x_i/a] \quad (11)$$

and $x_i = k_i/M$. Since $x_B > 1$ then $F[x_B] \simeq \frac{1}{2}x_B^3\exp[-2x_B/a]$. Thus ρ_{II} behaves as matter when averaged over many oscillations and its averaged pressure[3] is $\langle\overline{p_{II}}\rangle = (-B/a)\langle\rho_{II}\rangle \simeq 0$. Also since $x_H > x_B > 1$ then $F[x_B] \gg F[x_H]$.

Region "H:" This is our "tail" [2], defined as the part of the graph for which the frequency of the modes is smaller than the Hubble parameter, H. The functional behavior of the frequency with p is the same as in region II, therefore the averaged pressure expression for this region is the same as that of region II, that is, $\langle\overline{p_H}\rangle \simeq 0$. But the lower limit of integration k_H (or p_H) is given by the physical condition of the freeze-out of the modes by the expansion of the background universe

$$\omega_H[p_H] = H. \quad (12)$$

This region includes the modes from p_H to ∞ in the range in which ω is exponentially suppressed. The energy density of the tail is

$$\rho_H \simeq \frac{C}{a^3}\frac{k_H^3}{2M^3}\exp[-2k_H/aM]. \quad (13)$$

Notice that due to the freeze-out, the evolution of the k_H mode is highly nontrivial and thus corrections to the averaged pressure term $\langle\overline{p_H}\rangle \simeq 0$ will be important.

Modes in the tail, between p_H to ∞, behave differently from the other modes, since their time dependence is controlled by the Hubble expansion, Eq. (12). On the other hand, all modes with momenta $p \leqslant p_H$ redshift in the same way with the scale factor, towards decreasing values, i.e, the linear regime.[4] Nevertheless, these regions (0,I,II) also receive small modifications to their pressure term from the deep trans-Planckian regime. We show below that the modifications due to the p_H-defrosting effect are non-negligible and important only in the highly nonlinear regime, around p_H.

Now, we would like to estimate the corrections to pressure, Π_i, for all these regions, with the notation P_i for the effective modified pressure:

$$P_i \rightarrow \langle\overline{p_i}\rangle + \Pi_i, \quad (14)$$

where the index runs to $i = 0$, I, II, and H. The averaged unmodified "bared" pressure expressions, $\langle\overline{p_i}\rangle$, are

$$\langle\overline{p_{0,I}}\rangle \simeq \left(\frac{1}{3} - \frac{A}{a^2} + \cdots\right)\langle\rho_{0,I}\rangle, \quad (15)$$

$$\langle\overline{p_{II,H}}\rangle \simeq \left(-\frac{B}{a} + \cdots\right)\langle\rho_{II,H}\rangle. \quad (16)$$

The inverse power terms of a can be neglected and A, B are numerical constants related to the averaging (see the Appendix for details).

[3] See the Appendix for details of averaging.

[4] Modes in the linear regime are referred to as "normal modes."

In a similar manner to particle creation cases [17] in imperfect fluids [14,18], the highly nontrivial time dependence of the mode p_H and the transfer of energy between regions, due to the defrosting of this mode across the boundary p_H, give rise to pressure corrections in the fluid energy conservation law. The defrosting of the modes results in a time-dependent ''particle number'' for regions near p_H. From kinetic theory we know that this ''particle creation'' (the defrosting of the modes) gives rise to *effective viscous pressure modifications* [14,18]. The term Π_i denotes the effective viscous pressure modification to the ''bare'' pressure, $\langle \overline{p_i} \rangle$.

The criterion we use for modifying $T_{\mu\nu}$ is that the Bianchi identity must be satisfied [19] with the new expressions for pressure,[5] P_i,

$$\Sigma_i[\dot{\rho}_i+3H(\rho_i+\overline{p_i}+\Pi_i)]=\Sigma_i[\dot{\rho}_i+3H(\rho_i+P_i)]=0, \tag{17}$$

with $i=0$, I, II, and H. Let us write this expression explicitly in terms of its energy and bare pressure components, and collect the contributions of regions 0 and I into one combined radiation energy, $\rho_R=\rho_0+\rho_I$:

$$\dot{\rho}_{II}+3H(\rho_{II}+\overline{p_{II}})+\dot{\rho}_R+3H(\rho_R+\overline{p_R})=-3H\Pi_{II}, \tag{18}$$

$$\dot{\rho}_H+3H(\rho_H+\overline{p_H})=-3H\Pi_H, \tag{19}$$

where $\overline{p_{II,H}}\simeq 0$, $\overline{p_R}\simeq 1/3$. So, we have imperfect fluids in regions II and H, and eventually their energy is transferred, due to the redshifting effect, to regions 0 and I, which is why these regions also receive pressure modifications. Nevertheless, the viscous pressure corrections to the ''radiation'' modes are very small since the energy and the volume of phase space occupied by them is very large (a^3 times larger the Planck size volume). These regions are in a nearly equilibrium situations (see the Appendix).

Let us find Π_i, in order to solve Eqs. (18) and (19). As explained, the presence of Π_i is due to the exchange of energy between the two regions, from the defrosting of the modes p_H at the boundary. This is directly related to the time dependence of the boundary p_H, which in turn is going to be controlled by the Hubble parameter H. In essence, there is an exchange of modes between regions ($R+$ II) and H. Although the specific number of particles[6] in each of these regions, N_{II} and N_H, is not conserved, their rate of change, in the physical FRW Universe, is related through the conservation of the total number of particles which contains both of these components,

$$\dot{N}_T=0. \tag{20}$$

Each component satisfies[7]

$$\dot{\mathbf{N}}_{\mathbf{II}}=\Gamma_{II}\mathbf{N}_{\mathbf{II}}, \tag{21}$$

$$\dot{\mathbf{N}}_{\mathbf{H}}=(3H-\Gamma_H)\mathbf{N}_{\mathbf{H}}, \tag{22}$$

where the ''decay rates'' of the regions Γ_i account for the rate of change in the number of their ''particles'' (modes), due to the defrosting effect.

The system is not yet in equilibrium. The change in the number of ''particles'' gives rise to the effective viscous pressure Π_i. Even prior to the freeze-out effects, that is, even for $\Gamma_{H,II}=0$, the short-distance modes in regions II and III were out of thermal equilibrium, due to their nonlinear frequency and group velocity $v_g\neq 1$.

The contribution terms to pressure, Π_i, are related to Γ_i through [14,18]

$$3H\Pi_H=-(\rho_H+\overline{p_H})\Gamma_H, \tag{23}$$

$$3H\Pi_{II}=-[(\rho_{II}+\overline{p_{II}})+(\rho_R+\overline{p_R})]\Gamma_{II}. \tag{24}$$

Therefore, Eq. (19) reduces to

$$\dot{\rho}_H+(3H-\Gamma_H)(\rho_H+\overline{p_H})=0, \tag{25}$$

which can be also recast as

$$\dot{\rho}_H=\frac{\dot{n}_H}{n_H}(\rho_H+\overline{p_H}), \tag{26}$$

where $n_H=NH/a^3$ is the ''particle'' number density for the region of modes from p_H to infinity. The flow of particles is described by $\mathbf{n_H}=n_H u_a$, with u_a the unit four-velocity vector of the fluid. Notice that since the group velocity in the H region is negative, particles in this region flow in a direction v_g, opposite the direction of their momenta, k.

The number of ''particles'' N_H contained in the tail regime, in its preferred frame, is given by

$$N_H\simeq C_H\int_{k_H}^{\infty}dk k^2\exp[-k/aM]\simeq C_H(aM)\times k_H^2\exp[-k_H/aM], \tag{27}$$

where $C_H=N|\beta_k|^2$ is a constant proportional to the Bogoliubov coefficient $|\beta_k|^2$, and we keep an overall normalization constant N for the sake of generality. We can now calculate the energy transfer, due to the defrosting of the modes k_H, between the tail region and region II from the balance equation for N_H, Eq. (22), where $\dot{N}_H$ is

[5]From here on we drop the $\langle\cdots\rangle$ notation and denote the averaged ''bare'' pressure simply by $\overline{p_i}$ instead of $\langle\overline{p_i}\rangle$.

[6]We are loosely using the term particle here to refer to the wave packets of the trans-Planckian modes, centered around a momenta p_i.

[7]Vector objects related to the flow direction of the fluid are denoted in bold letters, e.g., $\mathbf{N_i}=N_i u_a$, with u_a the unit four-velocity vector of the fluid and the corresponding modulus of this vector N_i. Notice that the factor $(3HN_H)$ in Eq. (22) is related to the fact that the preferred frame for the tail modes falls along comoving geodesics in the Universe.

$$\dot{N}_H \simeq C_H \int_{k_H}^{\infty} dk k^2 \left(\frac{k}{aM} \right) \exp[-k/aM]$$
$$- C_H k_H^2 \exp[-k_H/aM] \dot{k}_H$$
$$\simeq 3HN_H - C_H k_H^2 \exp[-k_H/aM](\dot{k}_H - Hk_H)$$
$$\simeq N_H \left[3H + \frac{k_H/aM}{k_H/aM - 1} \left(\frac{\dot{H}}{H} \right) \right]. \tag{28}$$

In the last line we have used the approximation in Eq. (27), and

$$\frac{\dot{p}_H}{p_H} = \frac{p_H/M}{1 - p_H/M} \left(\frac{\dot{H}}{H} \right), \tag{29}$$

derived from Eq. (12). When $p_H \gg M$ (which always holds), we have, for Γ_H,

$$\Gamma_H \simeq 3H + \frac{\dot{H}}{H} \simeq 3H \left(\frac{1 - w_{total}}{2} \right), \tag{30}$$

where we have defined

$$\frac{\dot{H}}{H} = -\frac{3}{2} H (1 + w_{total}), \tag{31}$$

with $w_{total} = \overline{p}_{total}/\rho_{total}$ referring to the effective equation of state for the total energy density. Therefore, when $w_{total} \rightarrow -1$ then Γ_H reaches its limit, $\Gamma_H \rightarrow 3H$. Γ_H cannot change anymore once this limit is reached because the Hubble constant and the mode p_H freeze to a time-independent value. Notice that Γ_H is positive for all equations of state $w_{total} \leqslant 1$ and thus it slows down the dilution of the tail with the scale factor. Thus, the increase in the number of particles N_H as given by Γ_H does not allow the energy density of the ‘‘tail’’ to redshift as fast as matter. This indicates that although initially small, ρ_H eventually will come to dominate the total energy density.

We can repeat the same procedure for the modes in regions 0, I, and II in order to obtain a closed equation for $\dot{\rho}_{R,II}$, similar to Eq. (25), i.e., given entirely in terms of $\rho_{R,II}$ and $\overline{p}_{R,II}$,

$$\dot{\rho}_{II,R} + (3H - \Gamma_{II})(\rho_{II,R} + \overline{p}_{II,R}) = 0, \tag{32}$$

where we have used Eq. (24).

Let us now try to relate Γ_H to Γ_{II}. The total number of not frozen particles, N_{II}, in the region from zero to k_H is given by

$$N_{II} \simeq C_H [4M^3 + Mp_H \omega_H]. \tag{33}$$

From the total balance equation for the particle number between the two regions, $(R+II)$ and region H in the comoving volume, we have $\dot{\mathbf{N}}_{\mathbf{total}} = 0$, where $\dot{\mathbf{N}}_{\mathbf{II}} = \Gamma_{II} \mathbf{N}_{\mathbf{II}}$ and $\mathbf{N}_{\mathbf{H}} = Mp_H^2 \exp(-p_H/M)\mathbf{u} = Mp_H H \mathbf{u}$. Thus

$$\Gamma_{II} = \frac{Mp_H H}{4M^3 + Mp_H H} (\Gamma_H - 3H), \tag{34}$$

and $N_T = N_{II} - C_H(Mp_H \omega_H) = 4C_H M^3$. In obtaining the scalar quantity for the number of particles N_T from their flow $\mathbf{N_T}$, the negative sign picked up in the second term in N_T is related to the fact that the flow of the tail’s defrosted modes is in the direction opposite to their momenta, due to their negative group velocity. Therefore, by plugging in the expression of N_{II} from Eq. (33), we get that in the limit $N_{II} \gg Mp_H H$ that Γ_{II} is smaller than Γ_H and negative, given by the expression

$$\Gamma_{II} = -(3H - \Gamma_H) \frac{p_H}{4M} \frac{H}{M}. \tag{35}$$

Since $p_H H^2 \ll M^2 H$ then $y = p_H H/(4M^2) \simeq O(H/M)$ is going to be much less than 1 for as long as the expansion is not dominated by the tail. From the condition $\omega_H[p_H] = H$, and the time evolution of the physical momentum p_H in Eq. (29), we have that $\dot{p}_H/p_H \rightarrow -\dot{H}/H$, when $p_H \gg M$. The exact value of y does not matter and it is small. Nevertheless, the pressure modification Π_{II} slightly increases the dilution of ρ_{II} as determined by the equation for $\dot{\rho}_{II}$. The tail domination case, when ρ_H becomes comparable to ρ_{II}, should be treated separately since $\Gamma_{II} \rightarrow 0$.

In this part we calculate the effective equation of state for all the regions, from the pressure expressions, $\overline{p}_i$, Π_i that were obtained in the previous section. Starting with region H, and using Eqs. (19), (23) and (30), we have

$$\frac{\dot{\rho}_H}{\rho_H} = (\Gamma_H - 3H)(1 + w_H) = -\frac{3H}{2}(1 + w_{total})(1 + w_H). \tag{36}$$

The ‘‘effective’’ equation of state for the tail can be read from this expression to be

$$1 + \tilde{w}_H = \frac{1}{2}(1 + w_{total})(1 + w_H), \tag{37}$$

with $w_H \simeq 0$. The time evolution for ρ_H is

$$\rho_H = \rho_H(0) \exp\left[-\frac{3}{2} \int (1 + w_{total})(1 + w_H) d \ln a \right]$$
$$= \rho_H(0) \exp\left[-3 \int (1 + \tilde{w}_H) d \ln a \right]. \tag{38}$$

During radiation domination, $w_{total} = 1/3$, then the effective equation of state for the tail is $\tilde{w}_H = -1/3$; for matter domination, $w_{total} = 0$, then $\tilde{w}_H = -1/2$; at the start of the accelerated expansion, $q = 0$, $\rho_{II} = \rho_H$, we have $w_{total} = -1/3$, $\tilde{w}_H = -2/3$; and finally, if the tail dominates, then the only solution to the Friedmann equation is given by $w_{total} \simeq \tilde{w}_H$, with $\tilde{w}_H = -1$. Therefore, the tail starts to behave as dark energy in only a short amount of time, when its equation of state $\tilde{w}_H$ closely approaches the limiting value, $\tilde{w}_H = -1$.

By the same procedure, we can now estimate the effective equation of state for ρ_{II} in terms of Γ_{II}:

$$\frac{\dot{\rho}_{II}}{\rho_{II}} = \Gamma_{II} - 3H = -3H\left[1 + \left(\frac{1-w_{total}}{2}\right)y\right]. \tag{39}$$

Thus the effective equation of state for region II, obtained from Eq. (39), is

$$1+\tilde{w}_{II} = \left(1 + y\,\frac{1-w_{total}}{2}\right)(1+w_{II}), \tag{40}$$

where $y=(p_H/M)(H/M)=O(H/M)$ and $w_{II}\simeq 0$.

The time evolution of the ρ_{II} energy density component is

$$\rho_{II} = \rho_{II}(0)\exp\left[-3\int (1+\tilde{w}_{II})d\ln a\right]. \tag{41}$$

In a radiation dominated universe $(H/M)\propto(1/a^2)$, and during matter domination $(H/M)\propto(1/a^{3/2})$. The point is that the correction y to the matter equation of state for region II is small at least for up to the equality time. During most of the history of the Universe, y goes as an inverse power of the scale factor a. So region II behaves much like matter. The special era at which the tail eventually dominates the expansion and H becomes a constant (at $p_H=$ const, $\tilde{w}_H = w_{total} = -1$) is discussed below in Sec. III.

Similarly, by repeating the same steps, from Eq. (18), it can be shown that the modifications to pressure for ρ_R, regions 0 and I, are very small indeed. Thus their effective equation of state remains very nearly that of radiation, $\tilde{w}_R = 1/3$. In order to avoid repetition, we will not carry out the calculation for the effective equation of state of ρ_R, since the procedure is essentially the same as that for ρ_{II}, and it results in an effective radiation equation of state

$$1+\tilde{w}_R = \left(1 + y\,\frac{1-w_{total}}{2}\right)(1+w_R). \tag{42}$$

III. THE ISSUE OF COINCIDENCE AND COMPARISON TO OBSERVATION

From the computation of the pressures $\overline{p_H}$ and Π_i, it is clear that the initial radiation is redshifted faster than the other components of the total energy density. We can ask at what time, t_{eq}, the ρ_{II} components of matter become comparable to radiation,

$$\rho_R \simeq \rho_{II} \simeq \frac{\rho_{total}}{2} = \frac{3}{2}H_{eq}^2 M^2, \tag{43}$$

with

$$\rho_{II} \simeq C p_B^3 \exp[-2p_B/M] \simeq \frac{3}{2}H_{eq}^2 M^2. \tag{44}$$

From the above equation we obtain p_B at t_{eq}:

$$p_B \simeq \frac{M}{2}\ln\left[\frac{2Cp_B^3}{3H_{eq}^2 M^2}\right]. \tag{45}$$

On the other hand, from Eq. (12) we have

$$p_H \simeq M\ln\left[\frac{p_H}{H}\right], \tag{46}$$

and comparing Eqs. (45) and (46), it is clear than $p_B(t_{eq}) < p_H(t_{eq})$, and therefore

$$\rho_H(t_{eq}) < \rho_{II}(t_{eq}). \tag{47}$$

So matter-radiation equality takes place well before the eventual "tail" domination. From the equations of state, $\tilde{w}_H$ and $\tilde{w}_{II}$, Eqs. (37) and (40), we have that ρ_{II} always dilutes faster than ρ_H. Thus the inequality in Eq. (A4) holds true not only at $t=t_{eq}$ but also at all earlier times, before t_{eq}. Generally, there may be other sources of matter and radiation in the Universe, besides the contribution from the trans-Planckian modes. Although these components would not be affected by the viscous pressure corrections, Π_i, their contribution should included in the Friedmann equation when determining the equality time, $a(t_{eq})=a_{eq}$. Since their effect on the expansion is well studied and known, here we focus our attention only on the role of the trans-Planckian modes.

Let us now estimate the time at which the tail takes over to dominate the expansion and address the issue of the cosmic coincidence. We start by determining the time a_{DE}, that we have

$$\rho_H(a_{DE}) = \rho_{II}(a_{DE}) = \rho_{total}(a_{DE})/2, \tag{48}$$

or in terms of the density parameters $\Omega_H(a_{DE}) = \Omega_{II}(a_{DE})$. From the Friedmann equation for the expansion and the relation of $\tilde{w}_H$ to w_{total} it is straightforward to determine that at $a=a_{DE}$ we have

$$\tilde{w}_H(a_{DE}) = -\frac{2}{3}, \quad w_{total}(a_{DE}) = -\frac{1}{3}, \tag{49}$$

and therefore

$$\frac{a_{DE}}{a_{eq}} = \left[\frac{\rho_{II}^{(0)}}{\rho_H^{(0)}}\right]^{2/3(w_{total}+1)} = \left[\frac{\rho_H^{(0)}}{\rho_{II}^{(0)}}\right]^{1/2}, \tag{50}$$

with $\rho_i^{(0)}$ the value of the ith component at equality time, a_{eq}. It is interesting to notice that $w_{total} = -1/3$ corresponds to the transition time at which the deceleration parameter

$$q \simeq \frac{1}{2}(3w_{total}+1) \tag{51}$$

changes sign and goes through zero. This means that acceleration starts at the same time, a_q, as the dominance of the tail, a_{DE}, i.e., $a_q = a_{DE}$. Using the Friedmann expansion law, we can find the solution for the scale factor a, after the time a_q to be

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{II}}{3M^2} + \frac{\rho_H}{3M^2} = \frac{H^2(a_q)}{2}\left[\left(\frac{a_q}{a}\right)^3 + \frac{a_q}{a}\right]. \quad (52)$$

Therefore,

$$\int_1^{a/a_q} \sqrt{\frac{a/a_q}{(a/a_q)^2+1}}\, d(a/a_q) = \frac{H(a_q)}{\sqrt{2}}(t - t_q). \quad (53)$$

This integral can be done exactly and it is convoluted. The important point is that it gives a power law accelerated expansion, $a/a_q \simeq t^n$ with $n \geqslant 2$. Clearly the tail is behaving as dark energy and it is dominating the expansion soon after a_q.

Let us understand physically what occurs around the time $a = a_q$, and why $a_q = a_{DE}$. As shown in Sec. II, due to the strong coupling of the tail evolution to the Hubble constant, H, and therefore to ρ_{total}, $\tilde{w}_H$ becomes increasingly negative with decreasing values of w_{total}, soon after a_{eq}. Thus the tail starts to behave like dark energy, dominates the expansion, and approaches its limiting value $\tilde{w}_H = -1$ only at late times, when other energy contributions to ρ_{total} become negligible. From Eqs. (30) and (35), when $\rho_{II} \simeq \rho_H$ we obtain $\Gamma_{II} \simeq -y\Gamma_H/2$. When the tail dominates, the only solution to the Friedmann equation is $\Gamma_H \rightarrow 3H$, and then $w_{total} = \tilde{w}_H \simeq -1$. This means that the tail dominates the expansion ($w_{total} = -1$) very quickly and around that time ρ_{II} has become nearly zero. Recall that in this estimation we assumed that around the time a_q, ρ_{II} is the only source of matter. The fast dilution of ρ_{II} as compared to ρ_H, due to the viscous pressure effects of Γ_H, is the reason for $q = 0$ occurring at the same time as the tail dominance, a_{DE}. With no other sources of matter, w_{total} almost immediately goes from $w_{total} = -1/3$ to $w_{total} = -1$. Thus a_{DE} has occurred very recently indeed. If we consider other matter contributions in the Friedmann equation, that are independent of ρ_{II}, Γ_i, then the time of the expansion between the start of the accelerated expansion a_q and the time of tail dominance over ρ_{total} (i.e., when $w_{total} = w_H \simeq -1$) becomes a bit longer. This time interval from a_q to the present is also the time interval for which the tail has acquired a dark energy equation of state, until it reaches its limiting value, $\tilde{w}_H = -1$. Therefore, *cosmic coincidence is explained naturally by the intrinsic time evolution* of the effective equation of state for the tail, $\tilde{w}_H$.

IV. SUMMARY

Models of nonlinear short-distance physics discussed recently in the literature [8–13,16] usually introduce a time-dependent frequency, at the level of the equation of motion for the field. As a result the Bianchi identity is generically violated which indicates that Einstein equations need to be modified in the high energy regime. It is difficult to do so without an effective Lagrangian description of the theory in the trans-Planckian (TP) regime.

We therefore used a kinetic theory approach, in order to estimate the short-distance modification to the cosmic fluid stress-energy tensor for the model of Ref. [2]. It is not clear to us whether this procedure determines the modifications in a unique unambiguous way, or whether fluid idealization, and the assumption that kinetic theory remains valid at such high wave number modes, is a good approximation. Nevertheless, we believe that without an effective Lagrangian, kinetic theory is the only available tool to determine some sensible results for the contribution that TP modes make in the long wavelength regime.

In a previous paper [2], we showed that the energy contribution of the tail modes is comparable in magnitude to the observed dark energy in the universe. In this work we calculated the effective equation of state, $\tilde{w}_H$, for these tail modes in order to address the cosmic coincidence issue and showed that the tail modes behave as dark energy only at late times.

The tail has an exponentially suppressed frequency and all the modes with $\omega < H$ are frozen-out by the expansion of the background Universe. However, due to the short-distance pressure modification, the tail does not always behave as dark energy. The highly nontrivial time dependence of tail's dominant mode p_H tracks the evolution of the total energy density ρ_{total} through its strong dependence on the Hubble constant, H. The dependence of p_H on H is given by the freeze-out condition.

As a result of the coupling of the p_H mode to ρ_{total}, the tail equation of state $\tilde{w}_H$ follows the evolution of w_{total}, such that $\tilde{w}_H \simeq (w_{total} - 1)/2$. For this reason, $\tilde{w}_H$ acquires increasingly negative values as w_{total} decreases, from radiation to matter. The tail has a slower dilution with the scale factor compared to the other components and it start to dominate the expansion and behave as dark energy in only a short amount of time only, from the time at which the deceleration parameter q changes sign at time a_q. From this point a_q, with $w_{total} = -1/3$, $\tilde{w}_H = -2/3$, and onwards, the tail drives the Universe into an accelerated expansion, and soon reaches its limiting value of $w_{total} \simeq \tilde{w}_H \simeq -1$ with $\rho_{total} \simeq \rho_H$. Therefore, cosmic coincidence in this model is explained naturally from the time evolution of the tail's $\tilde{w}_H = f[w_{total}]$. This is the most important result of this paper.

We mention here that we have implicitly taken the ratio of the different energy components in the Universe, say, at the time of big bang nucleosynthesis, to be the conventional one. This amounts to tuning the Bogoliubov coefficient when computing the energy density contribution of the TP modes, by tuning the parameter ϵ_1 to be of order unity. This is equivalent to requiring the height of the plot in Fig. 1 to be within a few orders of magnitude of the Planck mass during inflation. This is a reasonable assumption and not a severely tuned value.

The TDE model was motivated by the closed string theory of the Brandenberger-Vafa model [4]. Therefore its observational implications may be explored to be indirect string signatures. Some of the distinctive features of the TDE model are the predictions that[8] the change in sign of the deceleration parameter, $q=0$, occurs at the same time as the start of tail dominance, i.e., the time at which the tail energy is half of the total, $a_q=a_{DE}$; this accelerated expansion occurs in a very short amount of time; due to the viscous pressure effects, the matter contribution from the TP modes goes through a change of its equation of state such that it starts to decay faster than normal matter. The latter effect shortens the time the Universe takes to change the parameters from $w_{total}\simeq -1/3, q=0$ to the time when $w_{total}\simeq -1, \rho_{total} \simeq \rho_H$. To arrive at these numbers, we ignored other matter sources as well as the short-distance corrections to the equations of state $\langle w_i \rangle$ that present as inverse powers of the scale factor, $O(1/a^n)$. Perhaps these corrections may be important, in terms of delaying the time the tail takes to behave as dark energy, $\tilde{w}_H \simeq -1$. These features can be scrutinized in future work [20].

Note added. The results obtained in this paper for the tail equation of state, $\tilde{w}_H$, differ from those reported by Lemoine *et al.* [21]. The fact that we include the curvature term a''/a under the definition of the generalized frequency ω^2, while they keep the two contributions separate, is not the only source of discrepancy. There are fundamental differences between the two studies. Authors of Ref. [21] ignore the crucial effects of the out-of-equilibrium dynamics and the breaking of Lorentz invariance, by the nonlinear short-distance modes, when carrying out their calculation for ρ_H and p_H. In their approach these effects would be contained in the dynamics of the vector field u_μ described by a Lagrangian $\mathcal{L}_u$ for this field. Obviously, the choice of u_μ field dynamics and its Lagrangian $\mathcal{L}_u$ *strongly depend on the details of the short-distance nonlinear model considered*. The expression for $\mathcal{L}_u$ that these authors borrow from the Corley-Jacobson (CJ) model [7] is *consistent only with the dispersion function of the CJ model* for stationary backgrounds, since both $\mathcal{L}_{cor}$ and the terms given by $\mathcal{L}_u$ contain up to fourth order derivatives, together with the antisymmetric tensor $F_{\mu\nu}$. Therefore, $\mathcal{L}_u$ gives zero corrections to ρ_H, p_H when applied to the Friedmann-Lemaître-Robertson-Walker Universe (where clearly the antisymmetric tensor vanishes identically to $F_{\mu\nu}$) and hence, higher order counterterms $(u_\mu)^n$ in $\mathcal{L}_u$ are ignored, while at the same time in $\mathcal{L}_{cor}$ they have a series of all higher order derivative terms, up to $n\to\infty$. It is easy to check whether they break the energy conservation law and Bianchi identity by plugging in the energy conservation equation to their expression for ρ and p in their Eqs. (35) and (36):

$$\langle \dot{\rho} \rangle + 3H\langle (\rho+p) \rangle \neq 0, \tag{54}$$

which we suspect is due to the aforementioned reasons.

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[8]These predictions are in the absence of other matter sources.

APPENDIX

1. Distribution function for the linear, crossover, and trans-Planckian regime

We have a time translation Killing vector for future infinity that determines our outgoing positive frequency modes. Let us consider our Universe as an expanding box, with size $L=a/M$, filled with modes. Inside this box we have a smaller box with fixed size $l_p \simeq 1/M$ that determines the range for the trans-Planckian (TP) modes. There is a preferred frame attached to the small box (due to the breaking of Lorentz invariance by the short-distance modes) but Lorentz invariance is restored in the big box, the expanding Universe. Following along the arguments and derivation in Ref. [16] for the "particle" distribution function, it is straightforward to apply their expression to our model. Below we consider three regimes depending on the value of momentum p with respect to the cutoff scale M:

(i) $p/M \ll 1$, the "normal regime." In this regime the frequency of the modes becomes linear,

$$\omega[p] \simeq p e^{-p/M} \to_{p/M\ll 1} p. \tag{A1}$$

The wavelength of these modes is then $\lambda \simeq O(L) \gg l_P$.

(ii) $p/M \simeq O(1)$, the crossover or intermediate regime, during which the TP modes go from the "TP box" with fixed size $l_P = 1/M_P$ into the "big box" with size $L=a/M$. This process occurs due to the redshifting of the modes, $p_i = k_i/a$. Each mode p_i will crossover and become a "normal mode" at some time $a_i = k_i/M$.

(iii) $p/M \gg 1$, the TP regime, such that $\lambda \simeq 1/k \ll l_P = 1/M$.

We do not report the derivation of the distribution function since the reader can find it in detail in Ref. [16]. In what follows we apply it to our case, to lend support to our assumption that the TP modes, *modes in the small box with fixed size $l_p = 1/M$ with respect to the preferred frame*, are not in thermal equilibrium, while modes in the range of the linear regime (*modes in the big box with size L=a/M, where Lorentz invariance is restored, the FRW Universe*) are in nearly thermal equilibrium:

$$\mathcal{N}(\omega) = \left| \frac{(\omega_{in}) v_g(p_{in})}{(\omega_{out}) v_g(p_{out})} \right| \left| \frac{\beta_p}{\alpha_p} \right|^2. \tag{A2}$$

The index in (out) refers to the incoming (outcoming) modes as defined in Ref. [2]; v_g is the group velocity,

$$v_g(p) = \frac{d\omega}{dp}, \tag{A3}$$

and β_p, α_p are the Bogoliubov coefficients, which in our model do not depend on the momentum p [2]. The dispersed frequency is given in Eq. (4), and thus

$$|v_g(p_{in})| = \frac{\omega_{in}}{p_{in}} \left| 1 - \frac{p_{in}}{M} \right|, \tag{A4}$$

$$v_g(p_{out}) \simeq 1, \tag{A5}$$

since $\omega(p_{out}) \simeq p_{out}$. The phase velocity is defined as $v_c = \omega/p$. Therefore

$$\mathcal{N}(\omega) = \left|\frac{\beta_p}{\alpha_p}\right|^2 (e^{-p/M_P}) \left| e^{-p/M_P}\left(1 - \frac{p}{M_P}\right)\right|. \quad \text{(A6)}$$

The thermal distribution is immediately recovered in the limit $p/M \ll 1$, i.e., $v_g \to 1$ and $\omega(p) \to p$.

Now we can evaluate the distribution function for the different regimes above:

(i) In this case

$$\mathcal{N}_a(\omega) \simeq \left|\frac{\beta_p}{\alpha_p}\right|^2, \quad \text{(A7)}$$

as it should, since we recover in the *out* region the normal linear frequency for the modes, $p/M \ll 1$ and $v_g \simeq 1$.

(ii) In the intermediate crossover regime, $p/M \simeq O(1)$, we have

$$\mathcal{N}_b(\omega) \simeq \frac{C}{2} \frac{e^{-2}}{e^{\beta\omega} - 1}, \quad \text{(A8)}$$

where we have identified the inverse of temperature $\beta \simeq a$ and the linear term in p/M with

$$\beta\omega \simeq \frac{1}{1 - p/M} = a\frac{M}{aM - k}, \quad \beta \simeq a. \quad \text{(A9)}$$

Clearly in this regime $|v_g|$ goes to zero, and $\beta\omega$ goes to infinity. The spectrum is nearly thermal, however, $\mathcal{N}(\omega)$ goes to zero, since as seen from the TP box the group velocity of these modes as they approach $p \simeq M$ becomes zero; or as seen from the normal particles in the big box, these modes have a high frequency ($\omega \approx M$), thus they do not contribute very much to the energy of modes in (i).

(iii) However, in the TP regime the distribution function strongly deviates from that of thermal equilibrium, since in this case $v_g(in) \neq 1$, and the frequency is highly nonlinear:

$$\mathcal{N}_c(\omega) \simeq \frac{C}{2} |e^{-p/M}| e^{-p/M} \left|1 - \frac{p}{M}\right|, \quad \text{(A10)}$$

with $p/M \gg 1$. Nevertheless $\mathcal{N}_c(\omega) \to 0$ when $p/M \to \infty$, thus their contribution to the energy is suppressed. The suppression comes directly from the frequency $\omega(p) \simeq pe^{-p/M}$ in this case.

The volume element in momentum space, dV_P, for the dispersed "particles," whose world line intersects a hypersurface element $d\Sigma$ around x, having momenta in the range $(p, p + dp)$, is $dV_p = 2\delta(p_\mu p^\mu) dp^4$, where p is future directed. Based on the definition of Ref. [16] for the inner product of the fields ($\phi[^{out}_{in}], \phi[^{out}_{in}]$) and integrating over the entire mass shell, the three-volume in momentum space is given by

$$dV_3 = a^3 \left|\frac{1}{v_g}\right| d^3p, \quad \text{(A11)}$$

which is consistent with the quantum field theory expressions of currents and "particle" number densities, given in terms of creation and annihilation operators. The distribution function, Eq. (A10), justifies our assumptions of deviation from thermal equilibrium in the TP regime.

2. Averaging of the TP energies, pressure, and equation of state

We have shown that modes in the TP regime of such very short wavelength $\lambda_{TP} \ll l_P$ are out of thermal equilibrium. Thus we need to average their effective contribution to the energy and pressure over many wavelengths, in order to obtain a nearly thermal, large scale state. This averaging is done over many wavelengths since, clearly, scales of cosmological order that are of interest to us are much much longer than any TP wavelengths. In what follows, we are interested in finding the averaged bare quantities $\langle \rho_i \rangle$ and $\langle \overline{p_i} \rangle$ before including any modification Π_i. The viscous pressure modifications, Π_i, occur due to the freeze-out and the change in the number of particles and are accounted for separately in Sec. II. Therefore region II will be grouped together with region H, since both have a highly dispersed TP frequency and, for the moment, we ignore the freeze-out corrections. Approximately we can write the energy of regions (0+I) and (II+H) in one compact form to avoid repetition:

$$\rho = p^4 \Theta(M - p) + \frac{M}{2} p^3 e^{-(p/M)\Theta(p-M)}, \quad \text{(A12)}$$

where $\Theta(p - M)$ is the unit step function that takes the value $\Theta(p - M) = 1$ for $p > M$ and zero otherwise. Clearly, for modes with $p/M \gg 1$ we get ρ_{II} (ρ_H). And for modes $M > p$ we get the radiation energy density of the (nearly) linear modes.

Using the energy conservation law (while ignoring the freeze-out effects)

$$\rho + \overline{p} = \frac{p}{3}\frac{d\rho}{dp} = \frac{p}{3}\left(\frac{d\rho_1}{dp} + \frac{d\rho_2}{dp}\right) = (\rho_1 + \overline{p_1}) + (\rho_2 + \overline{p_2}), \quad \text{(A13)}$$

with

$$\rho_1 = \frac{M}{2} p^3 e^{-p/M}, \quad p > M, \quad \text{(A14)}$$

$$\rho_2 = p^4, \quad p < M. \quad \text{(A15)}$$

And from Eq. (A13) we find

$$w_1 = \frac{\overline{p_1}}{\rho_1} = -\frac{k_B}{3aM}\Theta(k_B - aM), \quad \text{(A16)}$$

$$w_2 = \frac{\overline{p_2}}{\rho_2} = \frac{1}{3}. \quad \text{(A17)}$$

It is clear from the previous section that since the (nearly) linear modes, ρ_2, are nearly in thermal equilibrium then we do not need to bother with the averaging procedure for them (one can, however, check to verify the result $\langle w_2\rangle = 1/3$). This is not the case for the modes with $p/M > 1$, since these short wavelengths are out of equilibrium. Let us calculate $\langle \overline{p_1}\rangle$, $\langle \rho_1\rangle$, and $\langle w_1\rangle$:

$$\langle \rho_1\rangle = \frac{\int_0^a \rho_1 a^2 da}{\int_0^a a^2 da} = 3\frac{\int_0^{a^*=k/M} \rho_1 a^2 da}{a^3} \simeq \frac{14M^4 a^{*3}}{16 e a^3}. \tag{A18}$$

Similarly,

$$\langle \overline{p_1}\rangle = \frac{\int_0^a \overline{p} a^2 da}{\int_0^a a^2 da} = 3\frac{\int_0^{a^*=k/M} \overline{p} a^2 da}{a^3} = -\frac{14a*}{20a}\langle \rho_1\rangle, \tag{A19}$$

$$\langle w_1\rangle = \frac{\langle \overline{p_1}\rangle}{\langle \rho_1\rangle} = -\frac{14a^*}{20a} \simeq -\frac{B}{a}, \tag{A20}$$

where in Planck units, $a^* = l_p$ and we have used the normalization that Planck length $l_p = 1/M = a^* = a(t_P) = 1$. The time a^* corresponds to the crossover time at which the wavelength of the TP modes is on average of order of the size of the small box, $p(a^*) = M$. This time scale of order of the Planck box is much smaller than scales of cosmological interest, $L = a/M$. External observers bound to the large scale, the Lorentz invariant Universe with size $L = a/M$ and in thermal equilibrium, ''feel'' the energy and pressure contributions from the TP modes given by Eq. (A10).

Of course the real equation of state for these modes is given by their effective equation of state, $\tilde{w}_i$, Sec. II, that takes into account the relativistic kinetic theory modifications due to the freeze-out. In a similar manner, one can obtain $\langle w_2\rangle$ and in particular the numerical coefficient A for $\langle w_I\rangle, \overline{p}_I, \rho_I$.

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